Cosmological Perturbations and Numerical Simulations

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24th March 2010

arXiv:*0907.2917*, JCAP 0909:019

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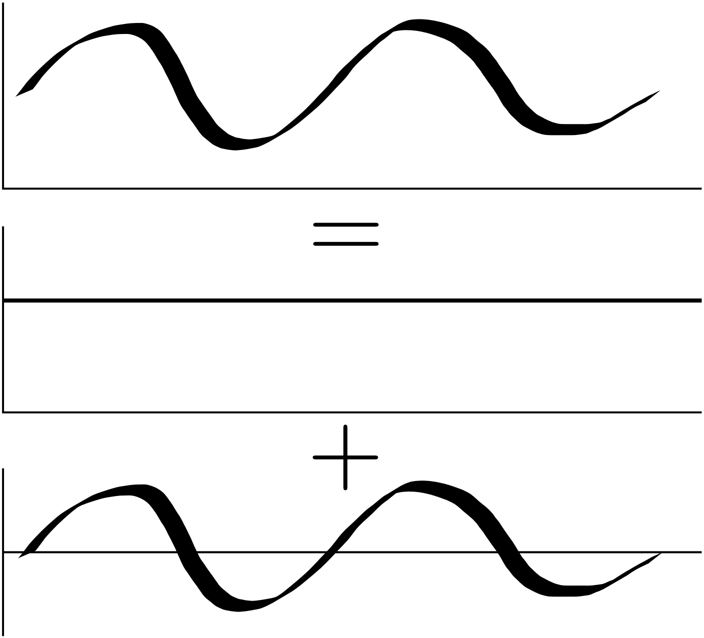
perturbations

Long review: Malik & Wands *0809.4944*

Short technical review: Malik & Matravers *0804.3276*



$$\mathbf{T}(\eta, x^i) = \mathbf{T}_0(\eta) + \delta\mathbf{T}(\eta, x^i)$$

$$\delta\mathbf{T}(\eta, x^i) = \sum_{n=1}^{\infty} \frac{\epsilon^n}{n!} \delta\mathbf{T}_n(\eta, x^i)$$

$$\varphi = \varphi_0 + \delta\varphi_1 + \frac{1}{2}\delta\varphi_2 + \ldots$$

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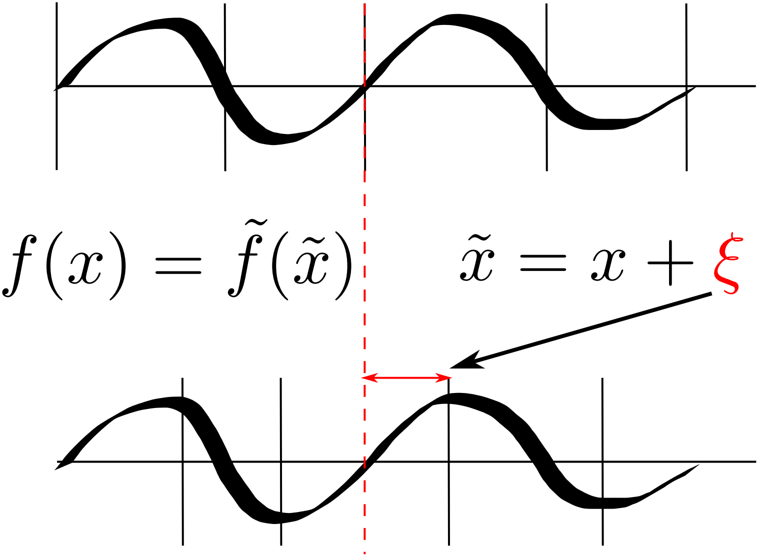
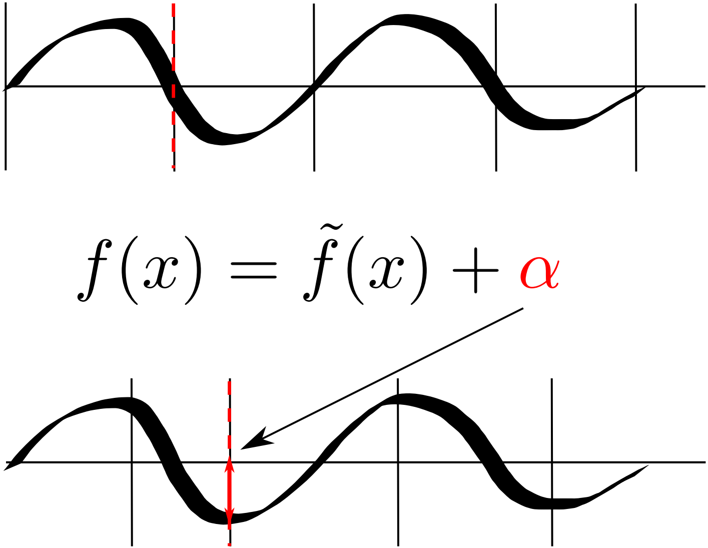
Gauges

- Background split not covariant
- Many possible descriptions
- Should give same physical answers!


$f(x) = \tilde{f}(\tilde{x})$
$\tilde{x} = x + \xi$


$f(x) = \tilde{f}(x) + \alpha$

First order transformation

$$\xi_1^\mu = (\alpha_1, \beta_{1,}{}^{i} + \gamma_1^i)$$

$$\Downarrow$$

$$\widetilde{\delta\varphi_1} = \delta\varphi_1 + \varphi_0'\alpha_1$$

Perturbed FRW metric

$$g_{00} = -a^2(1 + 2\phi_1)\,,$$
$$g_{0i} = a^2 B_{1i}\,,$$
$$g_{ij} = a^2\left[\delta_{ij} + 2C_{1ij}\right]\,.$$

Choosing a gauge

- Longitudinal: zero shear
- Comoving: zero 3-velocity
- Flat: zero curvature
- Uniform density: zero energy density
- ...

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

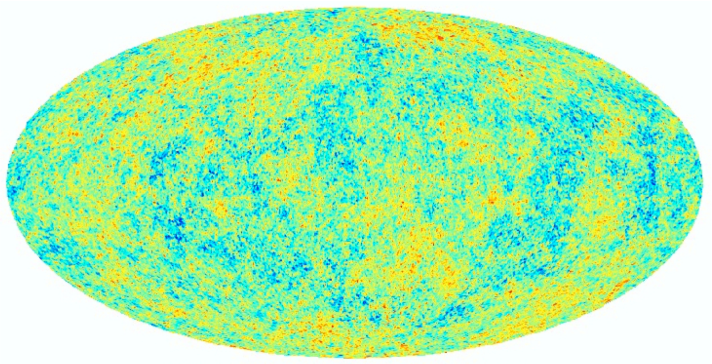
$$\Downarrow$$

Eqs of Motion

non-Gaussianity

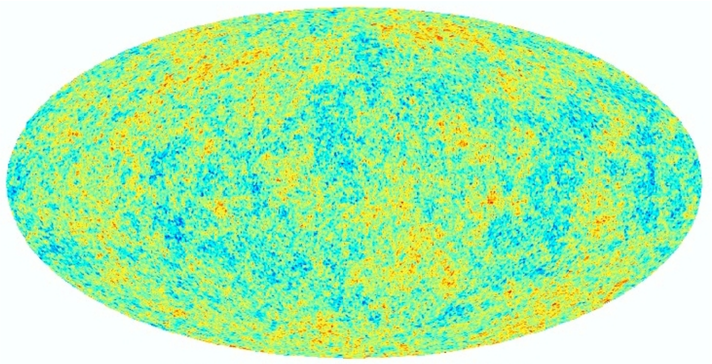
Some reviews: Chen *1002.1416*, Senatore *et al. 0905.3746*

Sim 1



Simulations from Ligouri et al, PRD (2007)

Sim 2



Simulations from Ligouri et al, PRD (2007)

Gaussian fields:

All information in

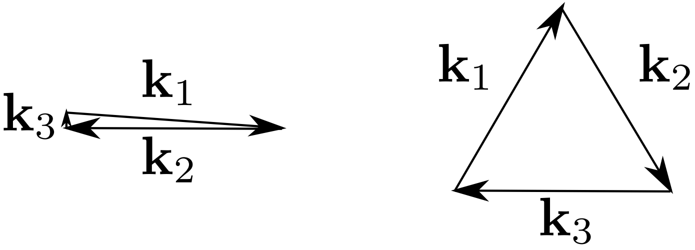
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\mathcal{P}_\zeta(k_1)\,,$$

where ζ is curvature perturbation on uniform density hypersurfaces.

$$\begin{aligned}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle &= 0\,,\\ \langle\zeta^4(\mathbf{k}_i)\rangle &= \langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle\langle\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle\\ &\quad+\ \langle\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle\langle\zeta(\mathbf{k}_4)\zeta(\mathbf{k}_1)\rangle\\ &\quad+\ \langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_3)\rangle\langle\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_4)\rangle\,.\end{aligned}$$

Bispectrum:

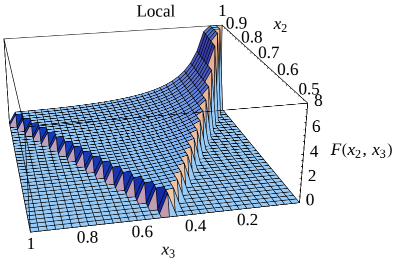
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

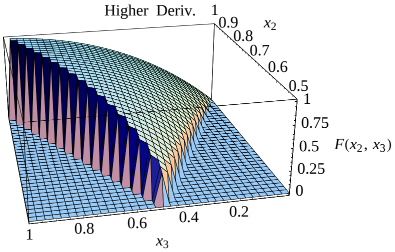


Local (squeezed) Equilateral

$$B(k_1, k_2, k_3) \simeq f_{\mathsf{NL}} F(x_2, x_3)\,,$$

$$x_i = k_i/k_1\,, \quad 1 - x_2 \le x_3 \le x_2\,.$$





Babich *et al. astro-ph/0405356*

WMAP7 bounds (95% CL)

$$-10 < f_{\mathrm{NL}}^{\mathrm{loc}} < 74$$

$$f_{\mathrm{NL}}^{\mathrm{loc}} > 1$$

rules out ALL single field inflationary models.

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One way of getting local f_{NL}

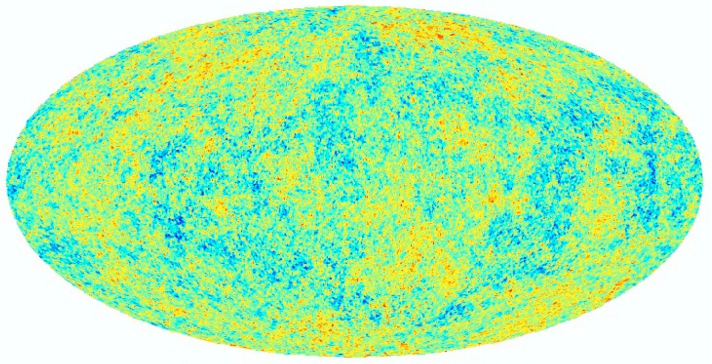
$$\zeta(\mathbf{x}) = \zeta_L(\mathbf{x}) + \frac{3}{5} f_{\mathrm{NL}}^{\mathrm{loc}} \zeta_L^2(\mathbf{x})$$

$$\frac{\Delta T}{T} \simeq -\frac{1}{5}\zeta\,, \qquad f_{\mathrm{NL}}^{\mathrm{loc}} > 0$$

$$\Downarrow$$

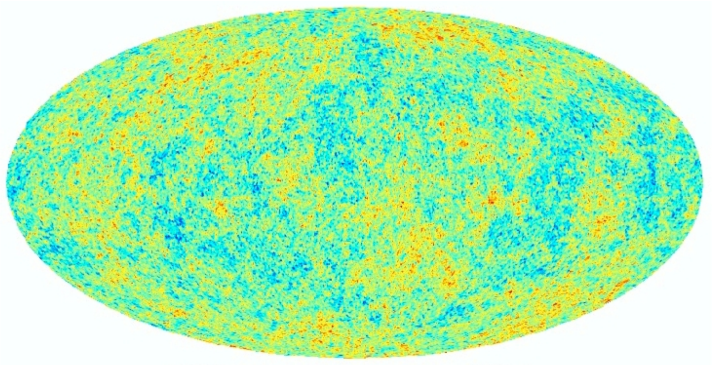
$$\Delta T < \Delta T_L$$

Sim 1: $f_{\mathrm{NL}} = 1000$



Simulations from Ligouri et al, PRD (2007)

Sim 2: $f_{\mathrm{NL}} = 0$



Simulations from Ligouri et al, PRD (2007)

code():

Paper: Huston & Malik *0907.2917*, JCAP

2nd order equations: Malik *astro-ph/0610864*, JCAP

Approaches:

- δN formalism
- Moment transport equations
- Field Equations

$$\varphi = \varphi_0 + \delta\varphi_1 + \frac{1}{2}\delta\varphi_2$$

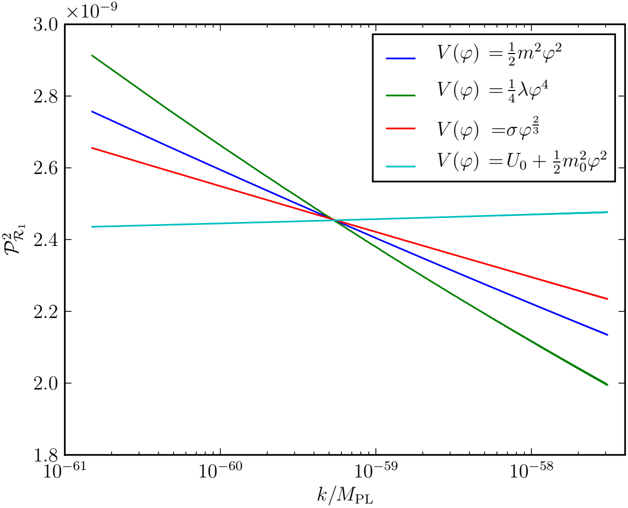
$$
\begin{aligned}
&\delta\varphi_2''(k^i) + 2\mathcal{H}\delta\varphi_2'(k^i) + k^2\delta\varphi_2(k^i) + a^2\left[V_{,\varphi\varphi} + \frac{8\pi G}{\mathcal{H}}\left(2\varphi_0' V_{,\varphi} + (\varphi_0')^2\frac{8\pi G}{\mathcal{H}}V_0\right)\right]\delta\varphi_2(k^i)\\
&+\frac{1}{(2\pi)^3}\int d^3p\,d^3q\,\delta^3(k^i-p^i-q^i)\left\{\frac{16\pi G}{\mathcal{H}}\left[X\delta\varphi_1'(p^i)\delta\varphi_1(q^i) + \varphi_0' a^2 V_{,\varphi\varphi}\delta\varphi_1(p^i)\delta\varphi_1(q^i)\right]\right.\\
&+\left(\frac{8\pi G}{\mathcal{H}}\right)^2\varphi_0'\left[2a^2 V_{,\varphi}\varphi_0'\delta\varphi_1(p^i)\delta\varphi_1(q^i) + \varphi_0' X\delta\varphi_1(p^i)\delta\varphi_1(q^i)\right]\\
&-2\left(\frac{4\pi G}{\mathcal{H}}\right)^2\frac{\varphi_0' X}{\mathcal{H}}\left[X\delta\varphi_1(k^i-q^i)\delta\varphi_1(q^i) + \varphi_0'\delta\varphi_1(p^i)\delta\varphi_1'(q^i)\right]\\
&\left.+\frac{4\pi G}{\mathcal{H}}\varphi_0'\delta\varphi_1'(p^i)\delta\varphi_1'(q^i) + a^2\left[V_{,\varphi\varphi\varphi} + \frac{8\pi G}{\mathcal{H}}\varphi_0' V_{,\varphi\varphi}\right]\delta\varphi_1(p^i)\delta\varphi_1(q^i)\right\}\\
&+\frac{1}{(2\pi)^3}\int d^3p\,d^3q\,\delta^3(k^i-p^i-q^i)\left\{2\left(\frac{8\pi G}{\mathcal{H}}\right)\frac{p_k q^k}{q^2}\delta\varphi_1'(p^i)\left(X\delta\varphi_1(q^i) + \varphi_0'\delta\varphi_1'(q^i)\right)\right.\\
&+p^2\frac{16\pi G}{\mathcal{H}}\delta\varphi_1(p^i)\varphi_0'\delta\varphi_1(q^i) + \left(\frac{4\pi G}{\mathcal{H}}\right)^2\frac{\varphi_0'}{\mathcal{H}}\left[\left(p_l q^l - \frac{p^i q_j k^j k_i}{k^2}\right)\varphi_0'\delta\varphi_1(k^i-q^i)\varphi_0'\delta\varphi_1(q^i)\right]\\
&+2\frac{X}{\mathcal{H}}\left(\frac{4\pi G}{\mathcal{H}}\right)^2\frac{p_l q^l p_m q^m + p^2 q^2}{k^2 q^2}\left[\varphi_0'\delta\varphi_1(p^i)\left(X\delta\varphi_1(q^i) + \varphi_0'\delta\varphi_1'(q^i)\right)\right]\\
&+\frac{4\pi G}{\mathcal{H}}\left[4X\frac{q^2 + p_l q^l}{k^2}\left(\delta\varphi_1'(p^i)\delta\varphi_1(q^i)\right) - \varphi_0' p_l q^l\delta\varphi_1(p^i)\delta\varphi_1(q^i)\right]\\
&+\left(\frac{4\pi G}{\mathcal{H}}\right)^2\frac{\varphi_0'}{\mathcal{H}}\left[\frac{p_l q^l p_m q^m}{p^2 q^2}\left(X\delta\varphi_1(p^i) + \varphi_0'\delta\varphi_1'(p^i)\right)\left(X\delta\varphi_1(q^i) + \varphi_0'\delta\varphi_1'(q^i)\right)\right]\\
&+\frac{\varphi_0'}{\mathcal{H}}\left[8\pi G\left(\frac{p_l q^l + p^2}{k^2}q^2\delta\varphi_1(p^i)\delta\varphi_1(q^i) - \frac{q^2 + p_l q^l}{k^2}\delta\varphi_1'(p^i)\delta\varphi_1'(q^i)\right)\right.\\
&\qquad\left.\left.+\left(\frac{4\pi G}{\mathcal{H}}\right)^2\frac{k^j k_i}{k^2}\left(2\frac{p^i p_j}{p^2}\left(X\delta\varphi_1(p^i) + \varphi_0'\delta\varphi_1'(p^i)\right)X\delta\varphi_1(q^i)\right)\right]\right\} = 0
\end{aligned}
$$

- ☑ Single field slow roll
- ☐ Single field full equation
- ☐ Multi-field calculation

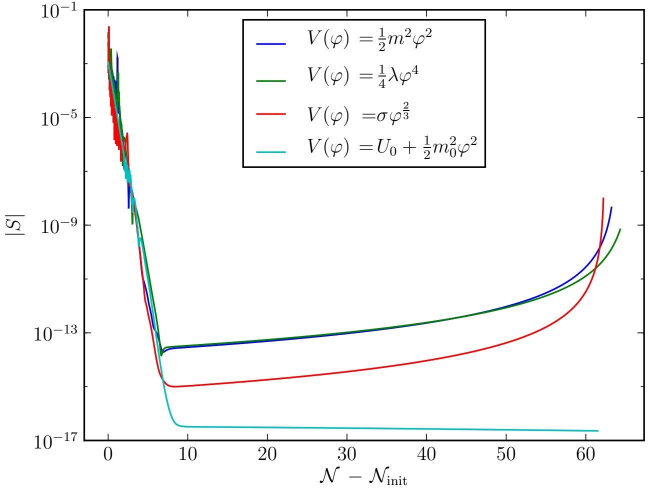
$$\int \delta\varphi_1(q^i)\delta\varphi_1(k^i - q^i)d^3q$$

```
code():
   1000+ k modes
   python & numpy
   parallel
```

Four potentials

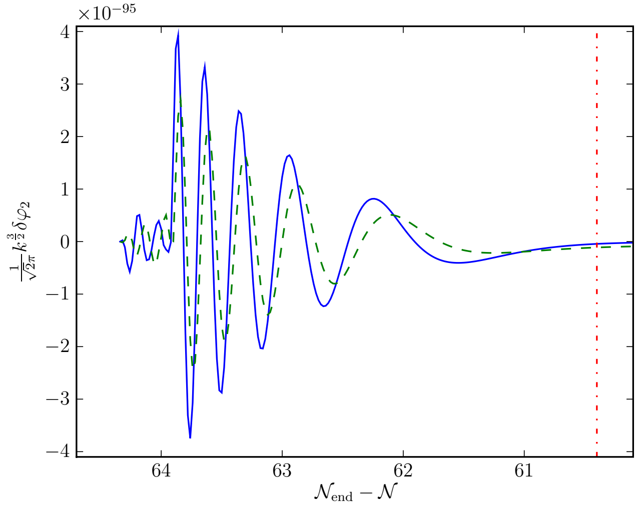


Source term



10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
$|S|$
0
10
20
30
40
50
60
$\mathcal{N} - \mathcal{N}_{\rm init}$
$V(\varphi) = \frac{1}{2}m^2\varphi^2$
$V(\varphi) = \frac{1}{4}\lambda\varphi^4$
$V(\varphi) = \sigma\varphi^{\frac{2}{3}}$
$V(\varphi) = U_0 + \frac{1}{2}m_0^2\varphi^2$

Second order perturbation



Future Plans:

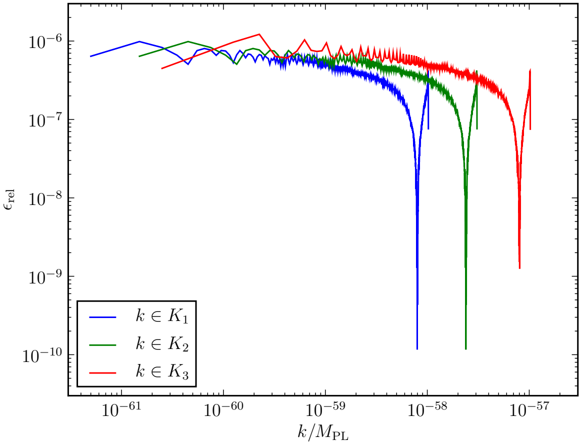
- Full single field equation
- Multi field equation
- Vector & Vorticity similarities
- Rework code for efficiency

Summary:

- Perturbations seed structure
- 2nd order needed for f_{NL}
- Numerically intensive calculation

$$I_{\mathcal{A}}(k) = \int \mathsf{d}q^3 \delta\varphi_1(q^i)\delta\varphi_1(k^i - q^i) = 2\pi \int_{k_{\mathsf{min}}}^{k_{\mathsf{max}}} \mathsf{d}q\, q^2 \delta\varphi_1(q^i)\mathcal{A}(k^i, q^i)\,,$$

$$\begin{aligned} I_{\mathcal{A}}(k) = -\frac{\pi\alpha^2}{18k}\Bigg\{ 3k^3 \Bigg[&\log\left(\frac{\sqrt{k_{\mathsf{max}} - k} + \sqrt{k_{\mathsf{max}}}}{\sqrt{k}}\right) + \log\left(\frac{\sqrt{k + k_{\mathsf{max}}} + \sqrt{k_{\mathsf{max}}}}{\sqrt{k_{\mathsf{min}} + k} + \sqrt{k_{\mathsf{min}}}}\right) \\ &+ \frac{\pi}{2} - \arctan\left(\frac{\sqrt{k_{\mathsf{min}}}}{\sqrt{k - k_{\mathsf{min}}}}\right)\Bigg] \\ &- \sqrt{k_{\mathsf{max}}}\Bigg[\left(3k^2 + 8k_{\mathsf{max}}^2\right)\left(\sqrt{k + k_{\mathsf{max}}} - \sqrt{k_{\mathsf{max}} - k}\right) \\ &\qquad + 14kk_{\mathsf{max}}\left(\sqrt{k + k_{\mathsf{max}}} + \sqrt{k_{\mathsf{max}} - k}\right)\Bigg] \\ &+ \sqrt{k_{\mathsf{min}}}\Bigg[\left(3k^2 + 8k_{\mathsf{min}}^2\right)\left(\sqrt{k + k_{\mathsf{min}}} + \sqrt{k - k_{\mathsf{min}}}\right) \\ &\qquad + 14kk_{\mathsf{min}}\left(\sqrt{k + k_{\mathsf{min}}} - \sqrt{k - k_{\mathsf{min}}}\right)\Bigg]\Bigg\}\,. \end{aligned}$$



$$K_1 = \left[1.9 \times 10^{-5}, 0.039\right] \,\mathrm{Mpc}^{-1}\,, \qquad \Delta k = 3.8 \times 10^{-5}\mathrm{Mpc}^{-1}\,,$$
$$K_2 = \left[5.71 \times 10^{-5}, 0.12\right] \,\mathrm{Mpc}^{-1}\,, \qquad \Delta k = 1.2 \times 10^{-4}\mathrm{Mpc}^{-1}\,,$$
$$K_3 = \left[9.52 \times 10^{-5}, 0.39\right] \,\mathrm{Mpc}^{-1}\,, \qquad \Delta k = 3.8 \times 10^{-4}\mathrm{Mpc}^{-1}\,.$$